STOP-BAND IMPROVEMENT OF RECTANGULAR WAVEGUIDE FILTERS USING DIFFERENT WIDTH RESONATORS: SELECTION OF RESONATOR WIDTHS

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Abstract— **Rectangular waveguide resonators having different widths can be mixed in order to improve the stop-band performance of band-pass filters. Two effective procedures for the choice of the resonator widths are presented and implemented to realise X-band 6-cavity filters which hold 30 dB of attenuation over a frequency range 40% wider than a standard filter. Theoretical and experimental results are shown and commented on.**

I. Introduction

Rectangular waveguide band-pass filters are widely used in microwave links. The high Q factor of rectangular waveguide resonators allows low loss and high selectivity. Unfortunately the stop-band performance is degraded by the re-resonances of the fundamental TE_{10} mode and the presence of higher order modes. As a consequence, a low-pass filter is often employed in order to recover the required level of attenuation at the frequencies of interest (usually the harmonic frequencies of the fundamental pass-band), adding cost and complexity to the manufacture.

A number of approaches have been proposed in order to improve the stop-band performance of such filters. The common idea is to modify the shape of each resonator so that the unwanted re-resonances are pushed up in frequency, while the fundamental resonance is kept at the pass-band frequency. A number of different ways exist in order to achieve this. The easiest one is to maintain the rectangular shape and simply vary the width of each resonator [1] [2]. In this way the analysis of the filter is relatively easy as the resonant modes of a rectangular waveguide resonator are preserved. Alternatively it is possible to employ ridged-waveguide resonators [3] or multiple inserts [4] [5], but the analysis and the physical realisation become more cumbersome.

No matter what approach is used, the stop-band performance of the filter can be further improved by carefully choosing the frequencies of the undesired re-resonances, that is by carefully designing the physical dimensions of each resonator. So far this choice has only been done on a trial basis in order to prove the solution being proposed.

In this contribution rectangular waveguide resonators with different widths will be discussed and the choice of the second resonances of the fundamental mode will be integrated in the usual filter design. In particular the solution with the re-resonances equally spaced over a defined frequency range will be investigated and simple design formulae for the resonator widths will be derived.

II. Waveguide resonator review

A waveguide section enclosed by two discontinuities (Fig. 1) is usually employed as the basic resonant structure of band-pass filters. The multimode equivalent circuit of a section of length l is shown in Fig. 2. For frequencies

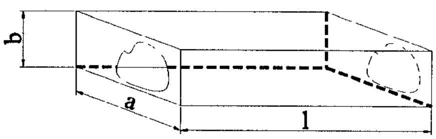

Fig. 1. Rectangular waveguide resonator.

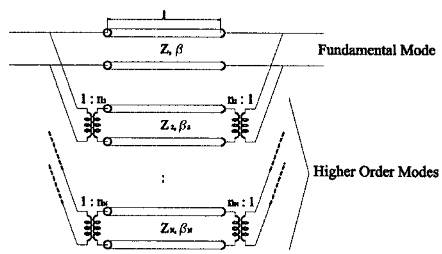

Fig. 2. Multimode equivalent circuit for a rectangular waveguide resonator.

such that the fundamental mode is the only mode capable to propagate, the excited higher order modes are localised around the discontinuities[1] and their effect can be modelled by shunt susceptances, as shown in Fig. 3, where the impedances have been normalised to the characteristic impedance Z of the equivalent transmission line of the fundamental mode. The transmission coefficient is given by

$$T = \frac{1}{\cos\vartheta - B_0 \sin\vartheta + j[B_0 \cos\vartheta + (1 - B_0^2/2)\sin\vartheta]} \quad (1)$$

[1]Provided that the length l is big enough to prevent interactions between adjacent discontinuities.

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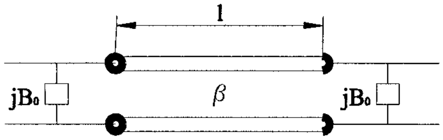

Fig. 3. Rectangular waveguide resonator mono-modal circuit.

and its magnitude is equal to unity (resonance), when the electrical length is

$$\vartheta = \arctan\left(\frac{2}{B_0}\right) + n\pi \qquad n = 0, 1, 2, \ldots \tag{2}$$

If the discontinuity apertures are small, then $B_0 \gg 2$ and the resonant electrical lengths are $\vartheta = \beta l \approx n\pi$; hence, given the fundamental resonant frequency f_0 and the resonator width a, the length l is easily determined by

$$l = \frac{\pi}{\beta} = \frac{a}{\sqrt{(f_0/f_c)^2 - 1}} \tag{3}$$

where $f_c = c/(2a)$ is the cut-off frequency of the fundamental TE_{10} mode. The second resonance occurs at frequency f_1 when $\beta(f_1)l = 2\pi$, that is $\beta(f_1) = 2\beta(f_0)$, from which

$$f_1 = \sqrt{4f_0^2 - 3f_c^2}. \tag{4}$$

Alternatively, given the resonant frequencies f_0 and f_1, the physical dimensions of the resonator a and l can be determined by

$$a = \frac{c}{2}\sqrt{\frac{3}{4f_0^2 - f_1^2}} \tag{5}$$

and (3), respectively.
Notice that the third resonance occurs at f_2, when $\beta(f_2) = 3\beta(f_0)$, that is

$$f_2^2 = 9f_0^2 - 8f_c^2 = 9f_0^2 - 8\left(\frac{4f_0^2 - f_1^2}{3}\right) \tag{6}$$

or

$$f_2 = \sqrt{\frac{8f_1^2 - 5f_0^2}{3}}. \tag{7}$$

III. Design

Resonators with different widths can be mixed to improve the stop-band performance of a band-pass filter [1] [2]. Obviously, all resonators are designed to have the fundamental resonant frequency at f_0, the centre frequency of the filter. The second resonant frequencies $f_1^{(i)}$ can be controlled by the widths a_i, and, if carefully chosen, the stop-band performance of the filter can be further improved. One possible criterion is to equally space the second resonances over a frequency range, which is defined by the two following constraints:

- since symmetric irises are employed, the first higher order mode to be excited will be the TE_{30} mode, whose cut-off frequency is $3f_c = 3c/(2a)$ (for a resonator of width a); the resonators will be designed to have the TE_{30} cut-off frequency greater than $2f_0$ (second harmonic rejection);
- all the third resonant frequencies $f_2^{(i)}$ will be kept greater than any second resonant frequencies $f_1^{(i)}$.

From the first constraint it follows that

$$a_{\max} = \frac{3}{4}\frac{c}{f_0} = 0.75\frac{c}{f_0}, \tag{8}$$

which defines the upper limit $f_{1\max}$ as

$$f_{1\max} = \sqrt{4f_0^2 - 3\left(\frac{c}{2a_{\max}}\right)^2} = \sqrt{\frac{8}{3}}f_0 = 1.633f_0. \tag{9}$$

The lower limit $f_{1\min}$ is determined by the second constraint which can be formulated as $f_{2\min} = f_{1\max}$; hence from (7) it follows that

$$\frac{8f_{1\min}^2 - 5f_0^2}{3} = \frac{8}{3}f_0^2 \tag{10}$$

or

$$f_{1\min} = \sqrt{\frac{13}{8}}f_0 = 1.275f_0 \tag{11}$$

and

$$a_{\min} = \sqrt{\frac{6}{19}}\frac{c}{f_0} = 0.56\frac{c}{f_0}. \tag{12}$$

Having defined the frequency range, the second resonant frequencies will then be equally spaced between $f_{1\min}$ and $f_{1\max}$ as follows:

$$\begin{aligned} f_1^{(1)} &= f_{1\min} \\ f_1^{(2)} &= f_{1\min} + \delta \\ &\cdots \\ f_1^{(n)} &= f_{1\min} + (n-1)\delta \end{aligned} \tag{13}$$

where n is the degree of the filter and

$$\delta = \frac{f_{1\max} - f_{1\min}}{n}. \tag{14}$$

The resonator widths a_i can be calculated by equation (5). Finally, the disposition of the resonators remains to be decided.
A 6-section filter with centre frequency $f_0 = 10$ GHz and a pass-band of 100 MHz was designed by following the above procedure starting from a Chebyschev prototype with $\epsilon = 0.04$; the resonator widths and the second and third resonant frequencies are summarised in Table I and the simulated transmission response is shown in Fig. 4. The filter is labelled *monotonic taper* because the resonators were ordered by their respective widths, that is a_1-a_2-a_3-a_4-a_5-a_6. The responses of other possible dispositions are shown in Figg. 5-6-7, where the *alternate taper*

TABLE I

WIDTHS AND RESONANT FREQUENCIES.

	a [mm]	f1 [GHz]	f2 [GHz]
1	16.85	12.75	16.33
2	17.43	13.35	17.56
3	18.11	13.94	18.76
4	18.91	14.54	19.93
5	19.86	15.14	21.08
6	21.03	15.73	22.21

refers to the order a_1-a_4-a_2-a_5-a_3-a_6, the *up-down taper* to the disposition a_1-a_3-a_5-a_6-a_4-a_2 and the *down-up taper* to a_5-a_3-a_1-a_2-a_4-a_6. By comparing the responses it can be said that the order of the resonators does not affect the response of the filter to a great extent. Perhaps the best overall response among these filters with equally spaced second resonances is obtained with the *monotonic taper.*

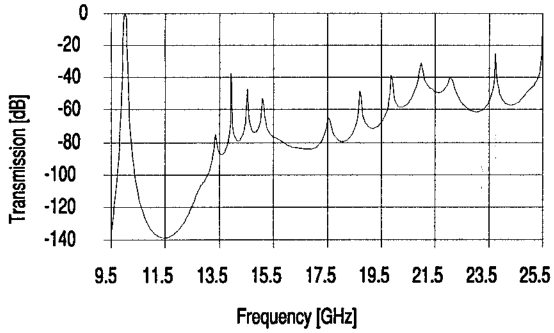

Fig. 4. Predicted transmission of the *monotonic taper* filter with equally spaced second resonances.

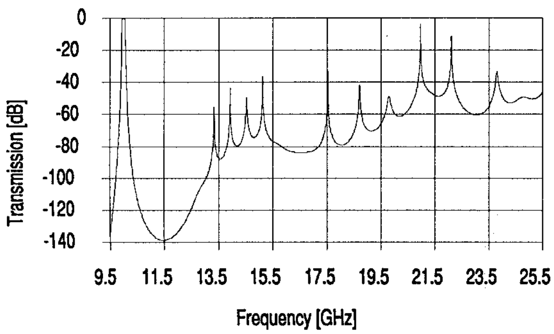

Fig. 5. Predicted transmission of the *alternate taper* filter with equally spaced second resonances.

Further improvements can be achieved by choosing only two second resonant frequencies (say f_1^a and f_1^b) and making half of the cavities re-resonate at one frequency and the other half at the second frequency. The choice of the two frequencies is based on a criterion similar to the previous one. In particular f_1^b can be chosen as (9) and f_1^a in such a manner that f_1^a, f_1^b and f_2^a are equally spaced, which

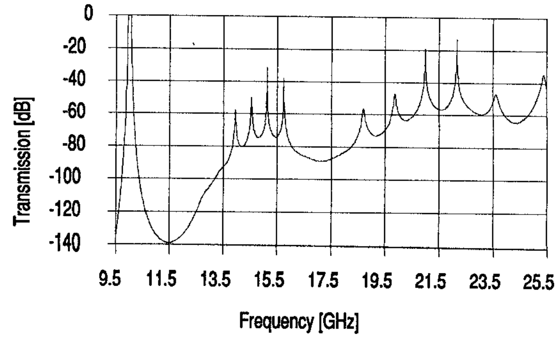

Fig. 6. Predicted transmission of the *up-down taper* filter with equally spaced second resonances.

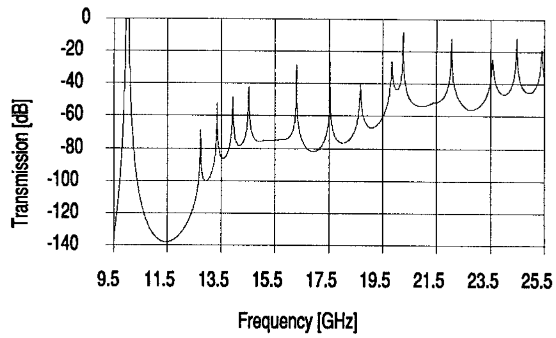

Fig. 7. Predicted transmission of the *down-up taper* filter with equally spaced second resonances.

results in

$$\begin{aligned} f_1^a &= 1.393 f_0 \\ f_1^b &= 1.633 f_0. \end{aligned} \qquad (15)$$

Again, having defined the two frequencies, the two widths a_1 and a_2 are determined by equation (5) and a number of different dispositions exist for the resonators. In particular two 6-section filters were designed with the same electrical characteristics of the previous filters. The predicted response of the filter with three adjacent resonators having width a_1 followed by three resonators having width a_2 (*3+3* filter) is shown in Fig. 8. The response of the

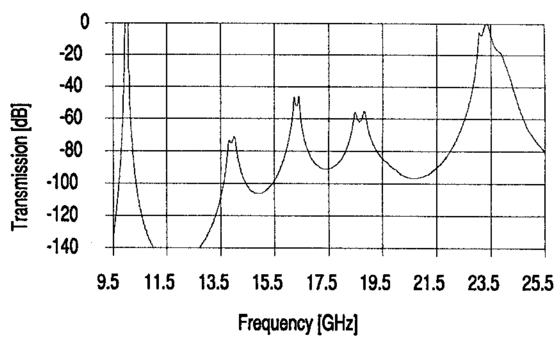

Fig. 8. Predicted transmission of the *3+3* filter.

filter with an alternate configuration (a_1-a_2-a_1-a_2-a_1-a_2) is shown in Fig. 9, which clearly indicates that a much better performance is obtained with the *3+3* filter.

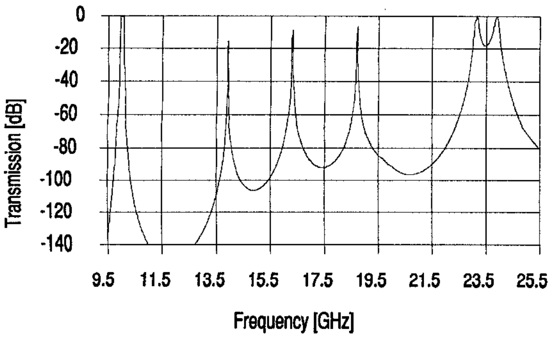

Fig. 9. Predicted transmission of the a_1-a_2-a_1-a_2-a_1-a_2 filter.

IV. Results

The *monotonic taper* filter and the *3+3* filter were realised and measured. The transmission responses are shown in Fig. 11 and Fig. 12, which are in good agreement with the predicted responses of Fig. 4 and Fig. 8 respectively.
Also a standard WR90 (a = 22.86 mm, b = 11.43 mm) filter was designed with the same electrical characteristics (6 sections; f_0 = 10 GHz; pass-band = 100 MHz). According to equations (4) and (7), the second and third resonant frequencies f_1 and f_2 were expected to be at 16.46 GHz and 23.58 GHz as indeed shown by the simulated response in Fig. 10. The standard filter holds an attenuation of

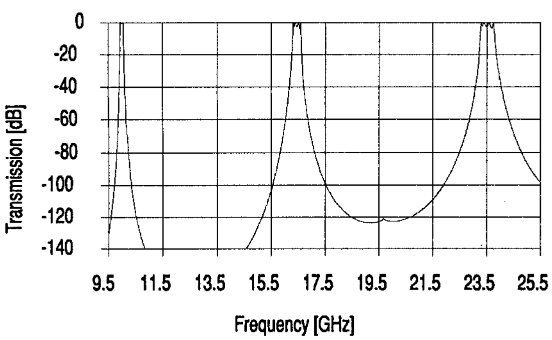

Fig. 10. Predicted transmission of the standard WR90 filter.

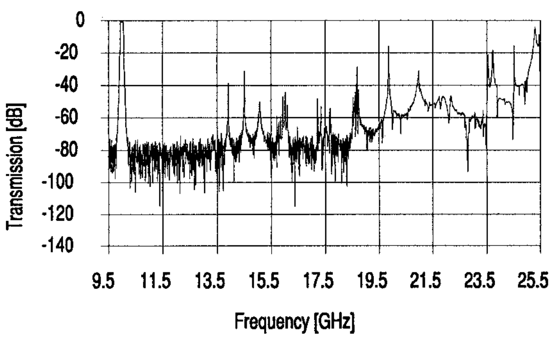

Fig. 11. Measured transmission of the *monotonic taper* filter with equally spaced second resonances.

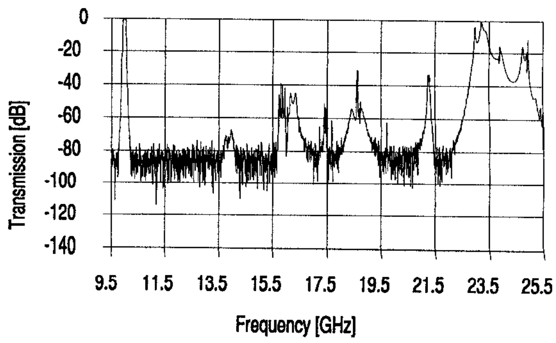

Fig. 12. Measured transmission of the *3+3* filter.

20 dB up to 16.3 GHz, while with the proposed *monotonic taper* filter an attenuation of 25 dB is achieved up to 19.8 GHz, which means that the stop-band has been extended by 20%. Moreover the *3+3* solution keeps an attenuation of 30 dB up to 22.8 GHz which, compared to the standard filter, translates in a 40% improvement.

V. Conclusions

A better stop-band performance of rectangular waveguide band-pass filters can be achieved by using resonators with different widths. In this paper two criteria for the choice of the cavity widths have been proposed. One has been based on equally spacing the second resonances over a defined frequency range and the achieved stop-band was 20% wider than a standard filter. The second solution has been based on having half of the cavities re-resonating at one frequency and the other half re-resonating at a second different frequency. In this case the stop-band improvement was even better and 30 dB of attenuation were held over a frequency range 40% wider than a standard filter. Finally experimental results confirmed the effectiveness of the proposed solutions.

Acknowledgement

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